Encoding Quantum Information in Transverse Modes of Light

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2005 June 09



Outline

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Polarization encoding is simple, but. . .

State space is spanned by $|\updownarrow\rangle$ and $|\leftrightarrow\rangle$.

- Easy state preparation (polarizers, PBS)
- Easy state manipulation (waveplates)
- Easy measurement (map polarization onto modes)

What about higher-dimensional systems?

- Better & faster QIP: bit commitment, fingerprinting, key distribution, etc.
- Methods of encoding make contact with more areas in physics: transverse modes and BECs


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Many Photons or Just One?

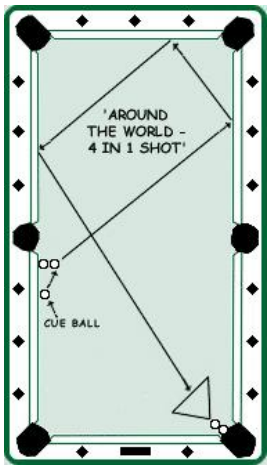
How should we encode the higher-dimensional system?

- Many photons: would only need $\sim \log(d)$ photons, but then they must interact
- If noninteracting, must use a smaller subspace (subsystem), e.g. permutation-invariant encoding
- Single photons: d "..." levels required

Encoding Higher Dimensions with Single Photons

In which physical degrees of freedom can we find enough levels? There are several possibilities:

- Direction: paths on (optical) table $\longmapsto$
- Frequency/wavelength: color encoding
- Time-of-arrival: pulse-train rdnoergi
- Transverse Modes: phase profile of propagating beams



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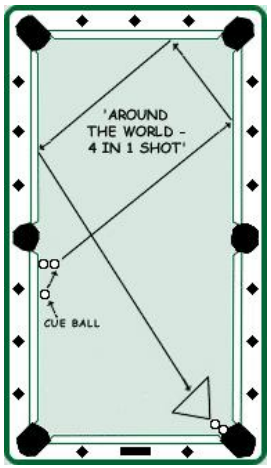

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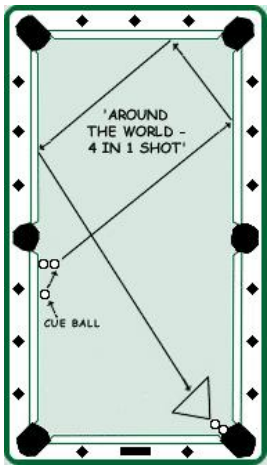

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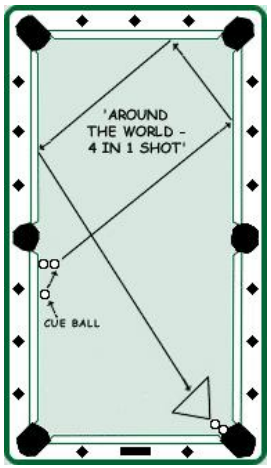

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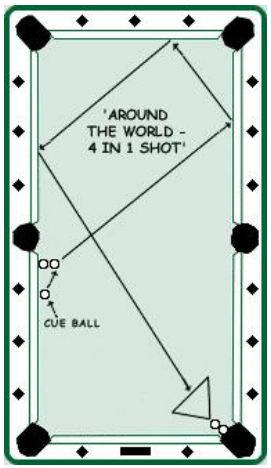


Paraxial Approximation

Transverse profile of beam changes slowly along the propagation direction

Monochromatic beam propagating along $\hat{z}$:

$$u(x,y,z,t) = v(x,y,z)e^{i(kz-\omega t)}$$

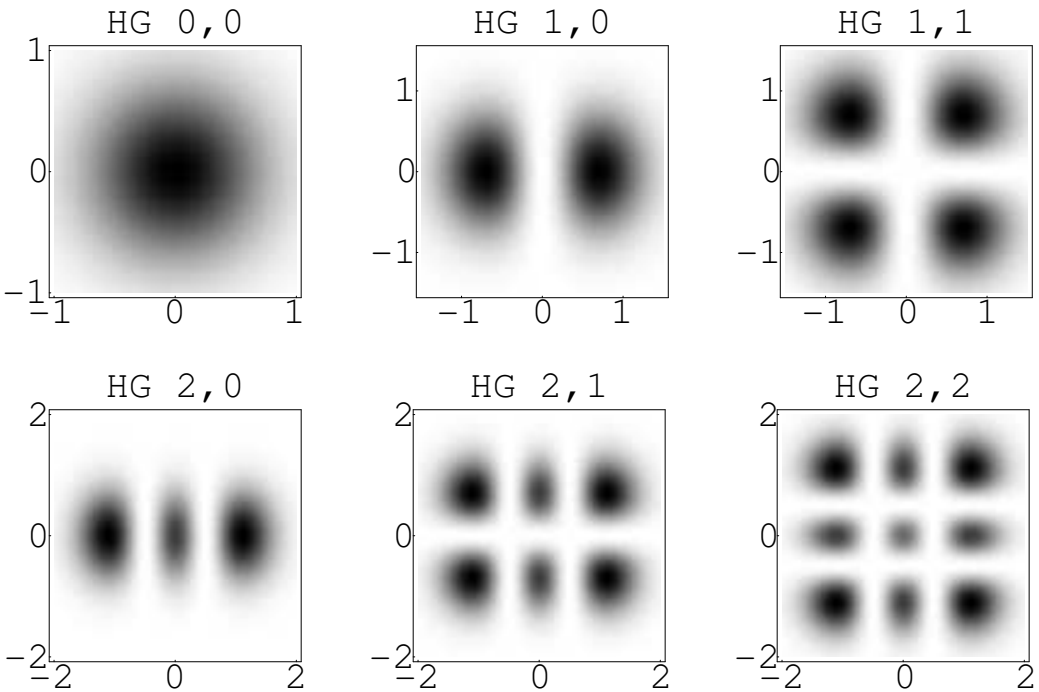
- $\frac{\partial v}{\partial z} \ll kv$
- $\frac{\partial^2 v}{\partial z^2} \ll k\frac{\partial v}{\partial z}$

$$\Rightarrow \quad \nabla_T^2 v + 2ik\frac{\partial v}{\partial z} = 0$$

Like 2D Schrödinger equation, with t replaced by z.

Simple Modes: Hermite-Gaussian

Using Cartesian coordinates we get Hermite-Gaussian modes
$\longrightarrow$ products of eigenstates of 1D harmonic oscillator.

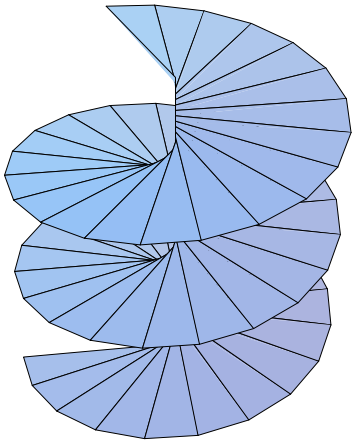


Footprint getting ever bigger, but better mode density than separate beams:
Area A supports d^2 T modes, d beams

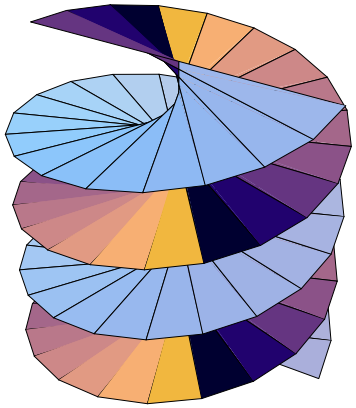
Orbital Angular Momentum Modes

Cylindrical coordinates $\rightarrow$ Laguerre-Gaussian modes $v_{p\ell}$

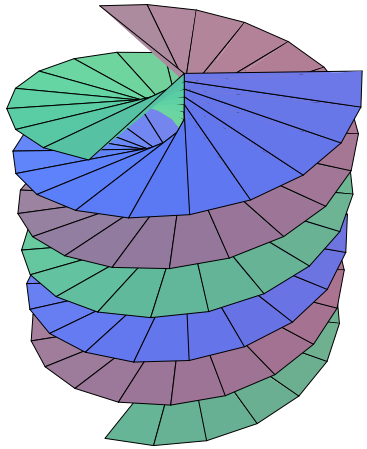
- p radial nodes
- spiral phase profile: $v_{p\ell}(r, \phi) \propto e^{i\ell\phi}$
- carry $\ell\hbar$ units of angular momentum per photon



$\ell = 1$



$\ell = 2$



$\ell = 3$

Fun with Fusilli

- Crypto without aligned reference frames, Spedalieri: quant-ph/0409057
- Couple OAM states to BEC vortices, Kapale & Dowling: quant-ph/0504130
- Toroidal traps for BECs, Wright, Arlt, and Dholakia: Phys. Rev. A **63**, 013608 (2001)
- Transfer angular momentum to trapped atoms, Phys. Rev. Lett. **91**, 093602 (2003)



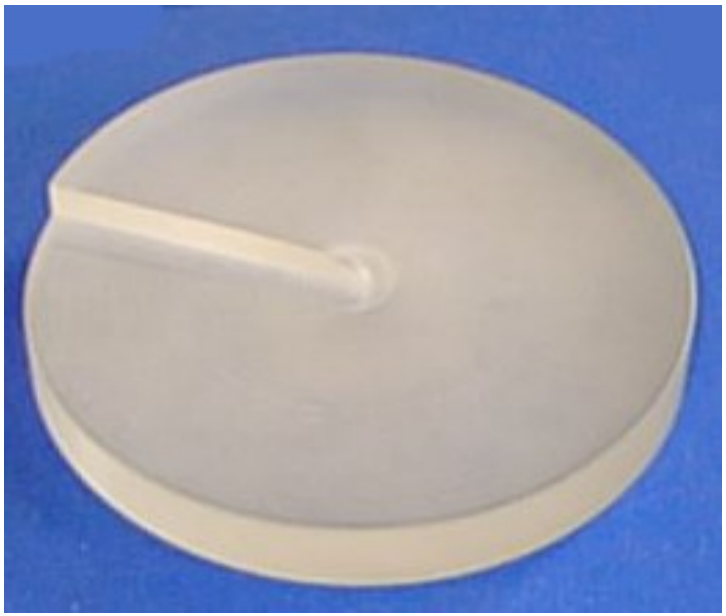
Generic Tasks of (Single-System) QIP

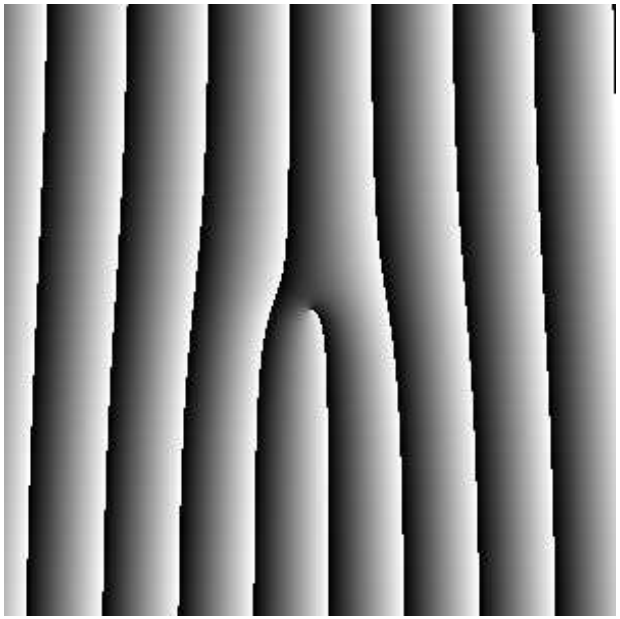
1. Prepare states, including superpositions
2. Manipulate states, i.e. effect arbitrary unitary operations
3. Make generalized measurements (POVMs)

Preparation of LG modes

Several Methods:

- Astigmatic Optics: HG modes $\leftrightarrow$ LG modes
- Spiral Phase Plate: increase/decrease ℓ
- Computer-Generated Holograms: arbitrary





Manipulating the Quantum State

Less important for prepare & measure communication schemes

- Good thing, too, because it's hard
- Can use holograms, but they have poor efficiency

One Method to Make Generic POVM Measurements

Perform measurement in four steps

1. Entangle transverse mode with propagation mode
2. Erase transverse mode information
3. Use beamsplitters and phase shifters to perform unitary operator (optical multiport)
4. Detect photons in each path

Exactly analogous to scheme for polarization

- Polarizing beamsplitter entangles polarization and direction
- Waveplate can erase polarization information
- Remaining steps identical

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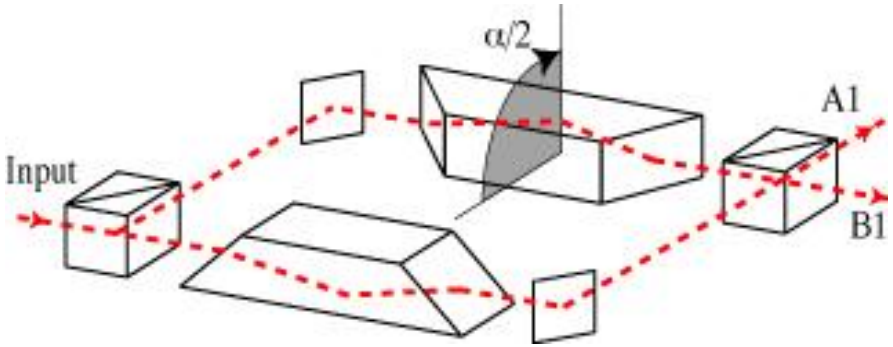
Mode Sorter

Entangle transverse mode with propagation mode

- Use Dove prism to flip phase profile of incoming beam
- Interfere beams flipped around different axes
- Equivalently: interfere image with rotated version
  (Dove prisms at relative angle $\alpha/2$ rotates image by α)



$\Rightarrow$ Sort modes according to odd or even ℓ-value

Mode Sorter

Separating more than odd from even

- Cascading several interferometers sorts powers of two: nth stage uses $\alpha = \pi/2^n$ — like Fourier transform
- To sort all modes, introduce phase shift into one arm: nth stage uses $\theta = -k\pi/2^n$ — this effectively makes the odds even.

$$\ell \mod 2^n \begin{cases} \ell \mod 2^{n+1} = k \\ \ell \mod 2^{n+1} = k + 2^n \end{cases}$$

- Drawbacks: Interferometric stability & optical table real estate. $d - 1$ interferometers required to sort d modes.
- Can also sort HG modes similarly, using GRINs


Erase Transverse Mode Information

- Transverse modes are sorted into different directional modes, but still entangled
- Use “waveplate” (hologram) to convert each mode to, say, Gaussian. This is the analog of a polarizing beamsplitter
- Drawbacks: holograms ruin efficiency of mode sorter. Maybe spiral phase plates?

Remainder of Measurement

Quantum information now encoded in directional modes:
Each mode is a basis state

- Implement desired unitary on these modes with beamsplitters and phase shifters
- This optical multiport implements unitary by acting one or two modes at a time
- Lastly, project onto basis states, e.g. find photon with photodetection


Entanglement of Angular Momentum

- Down-converted photons are transverse-mode entangled
- Shown by Zeilinger group in 2001: A. Mair *et al.*, Nature **412**, 313 (2001)
- Quantified by White group (UQ) in 2004: N. K. Langford *et al.*, Phys. Rev. Lett. **93**, 053601 (2004)