Dissecting Quantum Information

Classical conditions for quantum error-correction

TECHNISCHE UNIVERSITÄT DARMSTADT

Joseph M. Renes

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Prolog:

How do we design quantum information processing protocols?

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Answer: Begin with the end in mind (i.e. work backwards)

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quantum communication / entanglement or key distillation / state merging / etc.

use these to build other protocols

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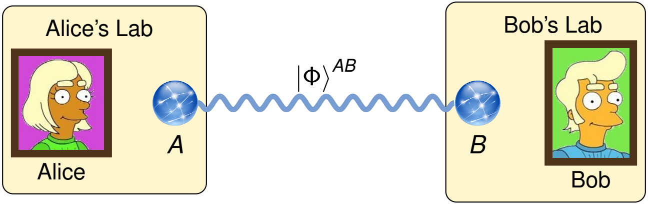

Take entanglement distillation. When can Bob recover entanglement from B?

two criteria: decoupling & crypto-based

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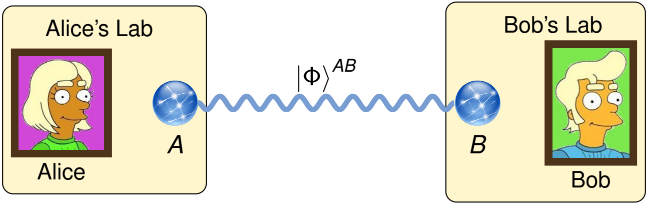

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Here I add **2 new approximate quantum error correction conditions** to this list

Outline

1. Existing approaches
2. Introduce new approaches
3. Some Details...
4. Applications & Open Questions

Part I

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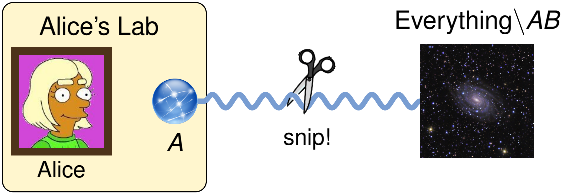
Existing Approaches

Current Approaches:

1. Decoupling

2. Crypto-based



Single quantum assumption: Bob can locally recover entanglement when A is mixed & decoupled from E. More technically:

if $\psi^{AE} \approx \tau^{A} \otimes \psi^{E}$ for $\tau^{A} = \mathbb{1}^{A}/\dim(A)$,

then there exists a $U^{B\to BCD}$ such that $U^{B\to BCD}\,|\psi\rangle^{ABE} \approx |\Phi\rangle^{AD}\,|\psi\rangle^{CBE}$

Great! Now we can start to design a protocol.

For any input state we just need to find a subspace of A which is decoupled and mixed.

Just pick a (sufficiently small) random subspace, it'll work with high probability...

Current Approaches:

1. Decoupling

2. Crypto-based

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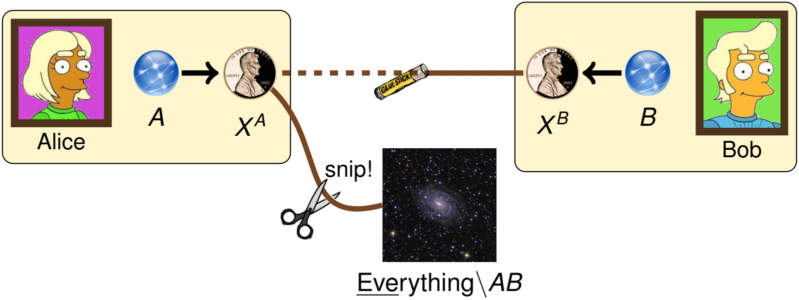


Two c-q assumptions: $U^{B \to BCD}$ exists when Bob knows X^A and Eve does not.

$$p_{\text{guess}}(X^A|B) \approx 1 \text{ and } p_{\text{guess}}(X^A|E) \approx 0 \text{ implies } U^{B\to BCD} |\psi\rangle^{ABE} \approx |\Phi\rangle^{AD} |\psi\rangle^{CBD}$$

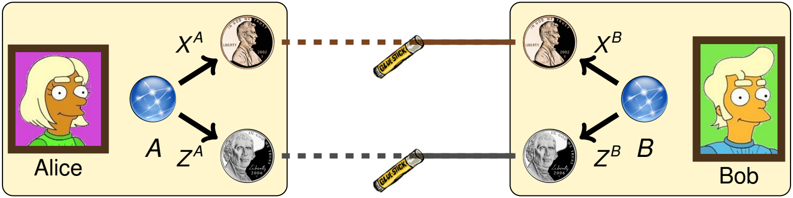
Part II

New Approaches

Two New Approaches

1. Complementary Classical Correlations for Bob

2. Complementary Classical Uncorrelations for Eve



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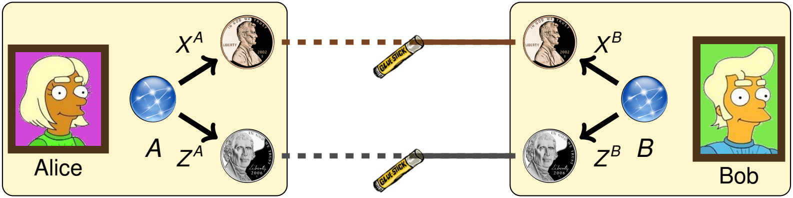

For an arbitrary ψ^{AB}, if

1. $p_{\text{guess}}(Z^A|B) \approx 1$ and
2. $p_{\text{guess}}(X^A|BC) \approx 1$,

then there exists a $U^{B\to BCD}$ with

$$U^{B\to BCD}\,|\psi\rangle^{ABE} \approx |\Phi\rangle^{AD}\,|\psi\rangle^{CBE}$$

(C: copy of Z^A // X, Z: Weyl-Heisenberg ops)

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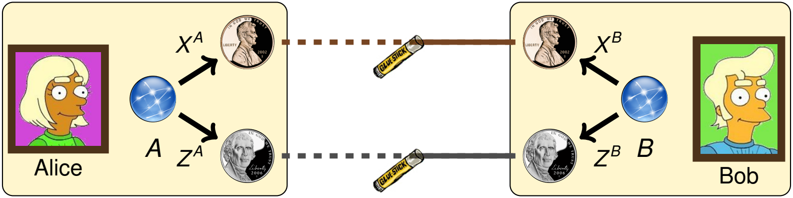

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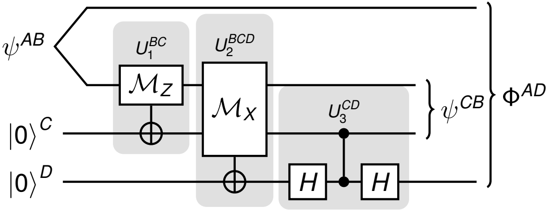

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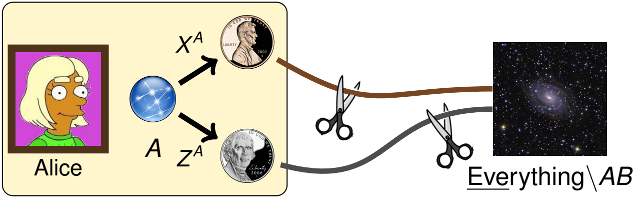

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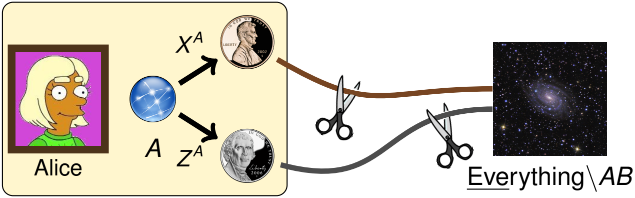

For an arbitrary ψ^{AB}, if

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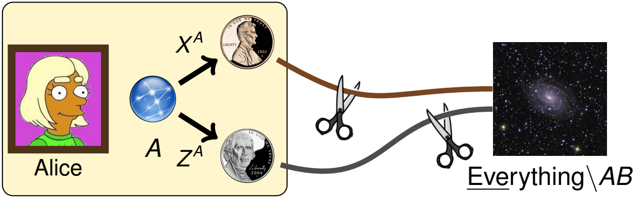

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(C is a copy of Z^A)

Duality: $p_{\text{guess}}(Z^A|B) \approx 1 \Leftrightarrow p_{\text{guess}}(X^A|CE) \approx 0$

Part III

Details: Bob's Decoder & Eve's Duality

A look at the details (Bob)

Building a decoder from Bob's measurements

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Start with $|\psi\rangle^{ABE} = \sum_z \sqrt{p_z}\,|z\rangle^A\,|\varphi_z\rangle^{BE}$ and $|\psi_Z\rangle^{ABCE} = \sum_z \sqrt{p_z}\,|z\rangle^A\,|z\rangle^C\,|\varphi_z\rangle^{BE}$

Assumptions: ♠ $p_{\text{guess}}(Z^A|B) = 1$ and ♣ $p_{\text{guess}}(X^A|BC) = 1$

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♠ implies there exists a $U_1^{B\to BC}$ taking $|\psi\rangle^{ABE}$ to $|\psi_Z\rangle^{ABCE}$

Why? Because Bob can measure B coherently to recover z, storing the result in C.

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♢ We can write $|\psi_Z\rangle^{ABCE} = \frac{1}{\sqrt{d}} \sum_x |\widetilde{x}\rangle^A \left(Z^{-x}\right)^C |\psi\rangle^{CBE}$

Insert $|z\rangle = \frac{1}{\sqrt{d}} \sum_x \omega^{-xz}\,|\widetilde{x}\rangle$ in $|\psi_Z\rangle$ and use $\omega^{-xz}\,|z\rangle = Z^{-x}\,|z\rangle$

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♣ implies $\exists\ U_2^{BC\to BCD}$ taking $|\psi_Z\rangle^{ABCE}$ to $|\psi_{ZX}\rangle = \frac{1}{\sqrt{d}} \sum_x |\widetilde{x}\rangle^A\,|-\widetilde{x}\rangle^D \left(Z^{-x}\right)^C |\psi\rangle^{CBE}$

Coherent mmt, same as before.

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♡ using a control phase from D to C (U_3^{CD}), we get $|\Phi\rangle^{AD} |\psi\rangle^{CBE}$

It just so happens that $|\Phi\rangle^{AD} = \frac{1}{\sqrt{d}} \sum_x |\widetilde{x}\rangle^A |-\widetilde{x}\rangle^D$.

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Bob's decoder is $U^{B\to BCD} = U_3^{CD} U_2^{BC\to BCD} U_1^{B\to BC}$. Also works approximately!!!!

Circuit for Bob's Decoder

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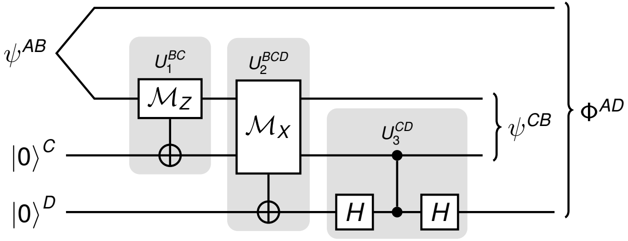

- Easy to prove that decoder also exists when $p_{\text{guess}}(X|B) \approx 1$.
- C takes care of correlations between X and Z; removes need to digitize errors
- C is also important for constructing optimal secret key & entanglement distillation protocols.
- Decoder of resulting protocol explicitly constructed

Details of Decoupling Eve

Duality of $p_{\text{guess}}(X^A|EC) \approx 0$ and $p_{\text{guess}}(Z^A|B) \approx 1$

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Fairly easy to see that $p_{\text{guess}}(Z^A|B) \approx 1 \Rightarrow p_{\text{guess}}(X^A|E) \approx 0$.

Bob's copy of Z^A dephases A in the Z basis; X^A is then random.

Converse is not true; consider state $|\psi\rangle^{ABE} = \frac{1}{\sqrt{2}}\left(|0\rangle + \mathrm{i}\,|1\rangle\right)^A |\varphi\rangle^{BE}$.

Both Bob and Eve ignorant: $p_{\text{guess}}(Z^A|B) = p_{\text{guess}}(X^A|E) = 0$.

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Two ways out: $p_{\text{guess}}(X^A|E) \approx 0 \Rightarrow p_{\text{guess}}(Z^A|B) \approx 1$ if either

1. Bob has a copy of X^A or
2. Eve has a copy of Z^A.

Case 1 leads back to the crypto scenario of entanglement distillation:
it says that Bob can recover entanglement when $p_{\text{guess}}(X^A|B) \approx 1$ and $p_{\text{guess}}(X^A|E) \approx 0$.

Case 2 corresponds to the case of a cq state for Alice and Bob:
Alice holds the classical variable Z^A due to dephasing from Eve.

More Duality (Eve)

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Start with $|\psi\rangle^{ABE} = \sum_z \sqrt{p_z}\,|z\rangle^A\,|\varphi_z\rangle^{BE}$ and $|\psi_Z\rangle^{ABCE} = \sum_z \sqrt{p_z}\,|z\rangle^A\,|z\rangle^C\,|\varphi_z\rangle^{BE}$

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♠ means we might as well start with $|\psi_Z\rangle^{ABCE}$, where Eve holds CE.

remember that $|\psi_Z\rangle^{ABCE} = \frac{1}{\sqrt{d}} \sum_x |\widetilde{x}\rangle^A \left(Z^{-x}\right)^C |\psi\rangle^{CBE} = \frac{1}{\sqrt{d}} \sum_x |\widetilde{x}\rangle^A\, |\vartheta_x\rangle^{BCE}$

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♣ means Eve's conditional marginals ϑ_x^{CE} are identical.
$\vartheta_x^{CE} = \vartheta_{x'}^{CE} \Rightarrow \vartheta_x^{CE} = \psi_Z^{CE}$ (just add the ϑ_x^{CE} together). In particular, $\vartheta_0^{CE} = \psi_Z^{CE}$, or $\psi^{CE} = \psi_Z^{CE}$

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♢ Uhlmann's theorem: $\exists\ U^{B\to MB}$ such that $U^{B\to MB}\, |\psi\rangle^{CBE} = |\psi_Z\rangle^{MBCE}$

MB belong to Bob, *CE* to Eve.

More Duality (Eve)

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♢ Uhlmann's theorem: $\exists\ U^{B\to MB}$ such that $U^{B\to MB}\,|\psi\rangle^{CBE} = |\psi_Z\rangle^{MBCE}$
MB belong to Bob, CE to Eve.

♡ Now apply $U^{B\to MB}$ to $|\psi_Z\rangle^{ABCE}$ and see what we get

Duality Cont'd

reminder: $|\psi_Z\rangle^{ABCE} = \sum_z \sqrt{p_z}\,|z\rangle^A\,|z\rangle^C\,|\varphi_z\rangle^{BE} = \frac{1}{\sqrt{d}}\sum_x |\widetilde{x}\rangle^A \left(Z^{-x}\right)^C |\psi\rangle^{CBE}$

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$$
\begin{aligned}
U^{B\to MB}\,|\psi\rangle^{ABE} &= U^{B\to MB}\,|\psi_Z\rangle^{ABCE} \quad (\spadesuit) && (1)\\
&= \frac{1}{\sqrt{d}}\sum_x |\widetilde{x}\rangle^A \left(Z^{-x}\right)^C U^{B\to MB}\,|\psi\rangle^{BCE} && (2)\\
&= \frac{1}{\sqrt{d}}\sum_x |\widetilde{x}\rangle^A \left(Z^{-x}\right)^C |\psi_Z\rangle^{MBCE} \quad (\clubsuit) && (3)\\
&= \frac{1}{\sqrt{d}}\sum_x |\widetilde{x}\rangle^A \left(Z^{-x}\right)^C \sum_z \sqrt{p_z}\,|z\rangle^M\,|z\rangle^C\,|\varphi_z\rangle^{BE} && (4)\\
&= \frac{1}{\sqrt{d}}\sum_{x,z} \sqrt{p_z}\,\omega^{-xz}\,|\widetilde{x}\rangle^A\,|z\rangle^M\,|z\rangle^C\,|\varphi_z\rangle^{BE} && (5)\\
&= \sum_z \sqrt{p_z}\,|z\rangle^A\,|z\rangle^M\,|z\rangle^C\,|\varphi_z\rangle^{BE} && (6)
\end{aligned}
$$

Thus, Bob can apply $U^{B\to MB}$ and then measure M to recover Z^A. Works approximately!!!!

Applications & Open Questions

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- Intuitive way to think about quantum information (processing)
  results in arXiv:1003.0703 & arXiv:1003.1150
- Construct optimal protocols with new approximate error-correction conditions.
  Secret-key / Entanglement distillation in PRA 78, 032335 (2008), *state merging* in LNCS 5906, 76 (2009), *mother of all protocols* coming soon.
- Proof of the quantum noisy channel coding theorem (with classical assistance).
- Quantum decoder explicitly constructed from classical decoders. Helpful in finding new quantum codes?
- Simple proof of HSW theorem (sending classical info over quantum channels), if we can convert these static results to the dynamic setting.
- Universal communication schemes? Quantum from 2x classical?
- One-shot versions?
- Weak string erasure... (quantum capacity !?!)