


Measurement-Based Quantum Computation in Realistic Spin-1 Chains

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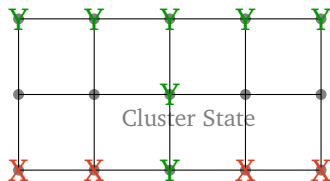
Perimeter Institute

Measurement-Based Quantum Computation

Single-site mmts on an entangled state

instead of

unitary gates on an unentangled state

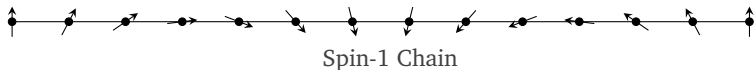


- New avenue in QC research — which states are useful?
- Practical implications — can we engineer these states?
- Connections to condensed matter — maybe ground states of realistic Hamiltonians are useful?

Explore the AKLT model

Outline:

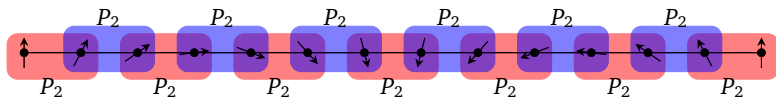
- 1 The AKLT model and how to perform single-qubit logic
Brief look at physical implementation
- 2 How to characterize computational ability of imperfect states
Numerical results: Computational ability is robust!
- 3 Improving performance: gate buffering and AKLT “distillation”
- 4 Summary and Outlook



Affleck-Kennedy-Lieb-Tasaki: Consider the nearest-neighbor Hamiltonian

$$H_{\text{AKLT}} = \sum_{j=1}^n (\vec{S}_j \cdot \vec{S}_{j+1}) + \frac{1}{3} (\vec{S}_j \cdot \vec{S}_{j+1})^2 \simeq \sum_{j=1}^n (P_2)_{j,j+1}$$

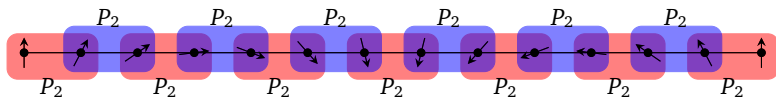
- ☞ Ground state is unique periodic BCs; 4fold degenerate open BCs
or $n \rightarrow \infty$ and $n < \infty$
- ☞ Gap to first excited state (Haldane conjecture, proven by AKLT)
- ☞ Ground state is a “valence bond solid” (VBS), frustration-free



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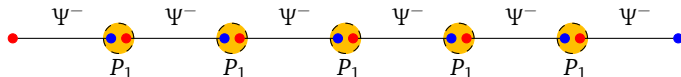
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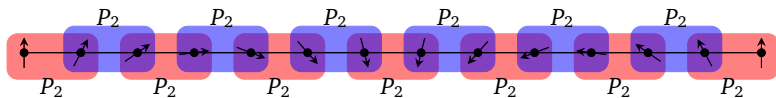


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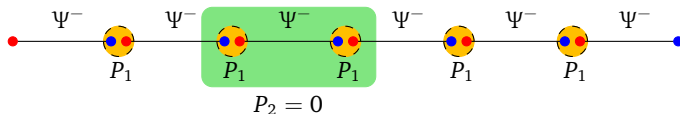




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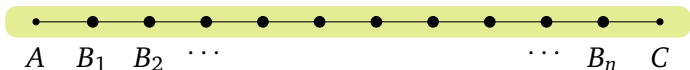
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VBS is the original matrix-product state

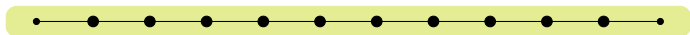
- Easy to work out using entanglement swapping relations
- For $|s\rangle \equiv |m_s = 0\rangle$, $s = x, y, z$ & Pauli operators σ_s (but with $\sigma_y = \sigma_x \sigma_z$):



$$|\mathcal{G}_{\text{open}}\rangle = \sum_{\{s_k\}} |s_1, s_2, \dots, s_n\rangle_B \otimes (\sigma_{s_n} \sigma_{s_{n-1}} \cdots \sigma_{s_1})_C |\Psi^-\rangle_{AC}$$

$$|\mathcal{G}_{\text{periodic}}\rangle = \sum_{\{s_k\}} \text{Tr} [\sigma_{s_n} \sigma_{s_{n-1}} \cdots \sigma_{s_1}] |s_1, s_2, \dots, s_n\rangle$$

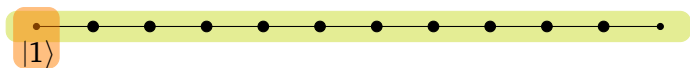
Single-qubit logic on the VBS



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➡ Chain encodes one logical qubit (C)

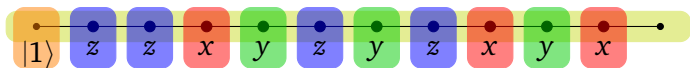
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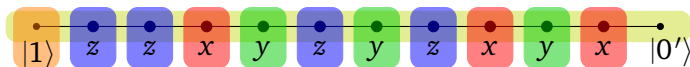
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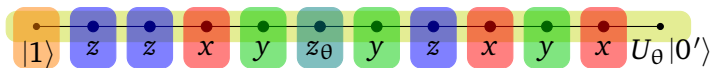
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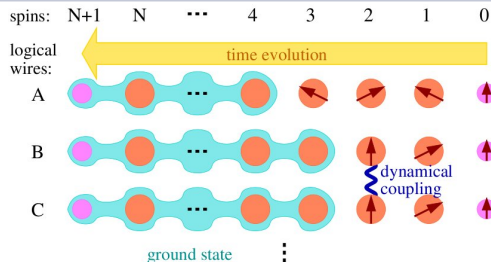
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“Byproduct” operators accumulate on C (change of basis)
- To rotate, measure in a rotated basis. Example: $R_{\hat{x}}(\theta)$

$$\{|x_\theta\rangle = |x\rangle, |y_\theta\rangle = \cos \frac{\theta}{2} |y\rangle + i \sin \frac{\theta}{2} |z\rangle, |z_\theta\rangle = i \sin \frac{\theta}{2} |y\rangle + \cos \frac{\theta}{2} |z\rangle\}.$$

Byproducts are σ_x , σ_y , and σ_z , respectively.

- Rotation is probabilistic, but heralded.

Brennen & Miyake scheme PRL 101, 010502 (2008).



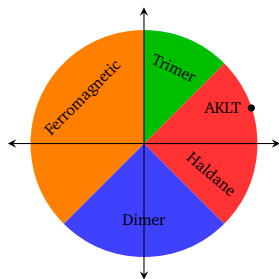
- 2D lattice of spins
- Individual chains as “quantum wires”
- Two-qubit logic (CPHASE) by dynamical coupling

Physical Implementation

Atoms or polar molecules in an optical lattice.

Gap protects against thermal noise

Imperfect Hamiltonians



☞ Can only engineer H_{AKLT} to some accuracy.

☞ Consider the Hamiltonians

$$H(\theta) = \sum_{j=1}^n \cos \theta (\vec{S}_j \cdot \vec{S}_{j+1}) + \sin \theta (\vec{S}_j \cdot \vec{S}_{j+1})^2.$$

☞ System is gapped in the Haldane phase (Haldane's conjecture)

☞ Computation should still be robust to noise.

☞ Using the same measurement scheme as for the AKLT state, how well can we perform single-qubit operations?

- ☞ Can in principle determine the output fidelity for any given mmt sequence and initial state. \Rightarrow But there are an exponential number!
- ☞ By the algebra of the Pauli group we can sum over the mmt sequences
- ☞ Work in the gate teleportation picture (Doherty & Bartlett arXiv:0802.4314v1 [quant-ph])
- ☞ $\langle X_A X_C \rangle' = \sum_{\vec{s}} \langle X_A X_C \rangle_{\vec{s}}$ with $\langle X_A X_C \rangle_{\vec{s}} = \langle \mathcal{G} | X_A P_{\vec{s},B} U_{\vec{s},C}^\dagger X_C U_{\vec{s},C} | \mathcal{G} \rangle$.
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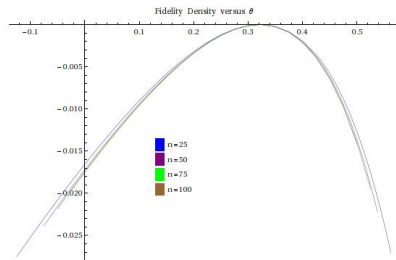
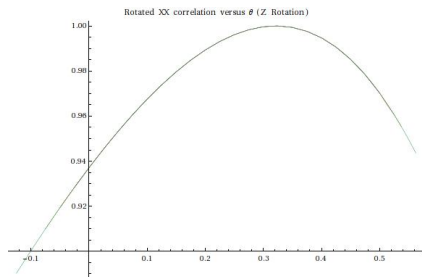
- $U_{\vec{s}}$ is a Pauli operator, just $X^{\#(x \text{ or } y)} Z^{\#(z \text{ or } y)}$.
- $(Z^{\#(z \text{ or } y)} X^{\#(x \text{ or } y)}) X (X^{\#(x \text{ or } y)} Z^{\#(z \text{ or } y)}) = (-1)^{\#(z \text{ or } y)} X$
- $(-1)^{\#(z \text{ or } y)} |s\rangle = e^{i\pi J_x} |s\rangle$.
- $\langle X_A X_C \rangle_{\vec{s}} = \langle \mathcal{G} | X_A \vec{V}_{x,B} P_{\vec{s},B} X_C | \mathcal{G} \rangle$ for $\vec{V}_{x,B} = \left(\bigotimes_{k=1}^n e^{i\pi J_{x,B_k}} \right)$.

☞ $\langle X_A X_C \rangle' = \langle \mathcal{G} | X_A \vec{V}_{x,B} X_C | \mathcal{G} \rangle$

- ➡ We only need to evaluate string operators $X_A \vec{V}_{x,B} X_C$ and $Z_A \vec{V}_{z,B} Z_C$.
- ☞ Similar to string operators $S_{x,j} \vec{V}_x S_{x,k}$ showing the “hidden order” of ground states in the Haldane phase (finite at any length, unlike usual spin correlations which decay exponentially.)
- ☞ Haldane conjecture (plus fixed BCs) $\Rightarrow \langle X_A X_C \rangle' = \langle Z_A Z_C \rangle' = 1$:
 - $X_A \vec{V}_{x,B} X_C$ is a global rotation by π about \hat{x} ;
 - H is rotationally-invariant and ground state unique.
- ➡ Near-AKLT ground states are also “good for (doing) nothing”.
- ☞ For single qubit rotations we need $\langle X_A R(\theta) X_C R(\theta)^\dagger \rangle'$, etc. Attempt rotations only a few times, average results.

Numerical results:

- ☞ Find ground states of the open/fixed chain using DMRG (actually use MPS form and imaginary time evolution)
- ☞ Localizable entanglement = 1 throughout Haldane phase, as expected.
- ☞ Rotation fidelity drops off slowly with theta (no computational phase)



- ☞ Nearby Hamiltonian: $H(\theta_{\text{AKLT}} + \epsilon) \approx H_{\text{AKLT}} + \frac{5}{6}\epsilon \sum_{j=1}^n \text{SWAP}_{j,j+1}$
 - ☞ Perhaps $|\mathcal{G}(\theta)\rangle = \alpha |\mathcal{G}_{\text{AKLT}}\rangle + \sum_{j=1}^n \beta_j \text{SWAP}_{j,j+1} |\mathcal{G}_{\text{AKLT}}\rangle$?
 - ☞ Can we improve fidelity by using swap-invariant mmt sequences?
- Doing nothing is already swap invariant: only # of x 's, y 's, z 's matters.
 - $R_z(\theta)$: Action of mmt sequence $|z\rangle |z_\theta\rangle |z\rangle = ZR_z(\theta) =$ invariant under swap
 - ➡ Measure outside spins first. If $|z\rangle |z\rangle$, proceed with middle spin. Otherwise measure middle spin in $|s\rangle$ basis.



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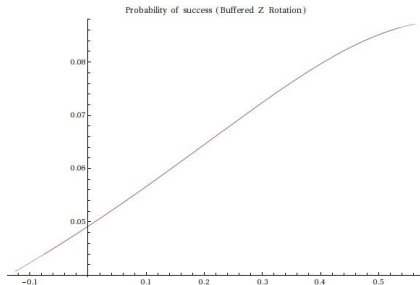
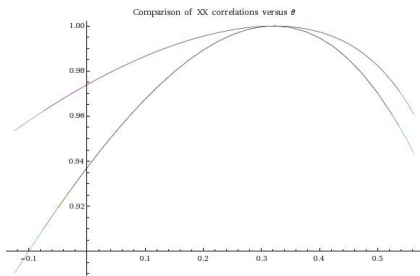
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- What about the $|y_\theta\rangle$ outcome?

- ☞ Action of $|z\rangle |y_\theta\rangle |z\rangle$ not swap invariant, but close: $\pm XZR_z(\theta)$ (minus as written, else plus)
- ☞ For $|\mathcal{G}(\theta)\rangle$ a superposition of $|G_{AKLT}\rangle$ and SWAP $|G_{AKLT}\rangle$, there are different paths to same properly-rotated state, but ± 1 phase.
- ☞ Fidelity should improve, but the overall probability of success is altered.



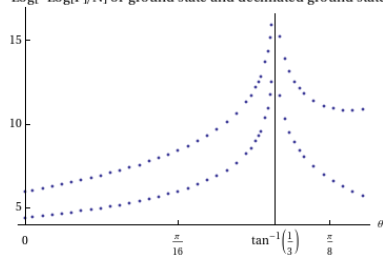
☞ Is there some simple transformation making the chain more AKLTlike?

- Should have something to do with repeated mmt outcomes
- Should respect the rotational symmetry

☞ Project neighbors onto spin-zero — $|\mathcal{G}_{\text{AKLT}}\rangle$ invariant!

- $|S = 0\rangle = \frac{1}{\sqrt{3}} (|xx\rangle - |yy\rangle + |zz\rangle)$
- $|\mathcal{G}\rangle = \sum_{\{s_k\}} |s_1, s_2, \dots, s_n\rangle_B \otimes (\sigma_{s_n} \sigma_{s_{n-1}} \cdots \sigma_{s_1})_C |\Psi^-\rangle_{AC}$

-Log[-Log[F]/N] of ground state and decimated ground state



☞ Project neighbors onto spin 0 or 1

☞ Project n-n-neighbors onto spin-zero (gate buffering)

☞ Probably many more

☞ Dimer state also a fixed-point.

Summary and Outlook

- ☞ Can efficiently characterize computational ability of spin-1 chains.
 - ☞ Near-AKLT ground states make good quantum wires.
 - ☞ Buffered measurements improve performance.
 - ☞ Simple projection operations make AKLT-like states even more so.
-
- ☞ Two-qubit gates (CPHASE)? Thermal noise?
 - ☞ Analytic results (perturbative or variational using swap states)?
 - ☞ Is the AKLT distillation just a curiosity?

Thanks for your attention!